Failure to Construct and Transfer Correct Representations across Probability Problems

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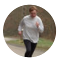

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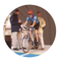

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Failure to construct and transfer correct representations across probability problems

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Abstract. Previous studies carried out on «purely random» situations (with dice or poker chips) show the difficulties encountered by people in such situations however simple they may be. In fact, in this type of situation prior knowledge guides spontaneous representations and the «errors» observed could be explained by the activation of «implicit models» which form the basis of erroneous representations. In the present research, our goal is to examine whether learners can construct adequate representations of the situation, and how these adequate representations can be transferred to a structurally isomorphic situation. 42 statistically naïve undergraduates were given five consecutive variants of an error prone probability problem. The first four problems involved geometric figures (two triangles and a square), two of the problems were structurally isomorphic (« equivalent ») and the other two had a complementary structure. The fifth problem, which involved poker chips (two reds and one white), was structurally isomorphic to the fourth geometric-figures problem. Our findings show that (1) Students did not realize that the four geometric-figures problems were structurally related, with some variants inducing higher proportions of correct answers than other variants. It shows that people reason differently about structurally isomorphic problems, and fail to realize the complementary status of situations. (2) Students who performed correctly on the four geometric-figures problems also solved correctly the poker-chips problem. (The didactic implications of the results in teaching mathematical concepts are discussed.)

These results appear to have some significant implications in teaching mathematical concepts.

INTRODUCTION

The aim of this research is to study spontaneous representations and their evolution in «purely random» situations, e.g. situations where samples are drawn at random from a set. The experimental work presented below is in keeping with a research program which insists on the value, if not the necessity, of attempting to act upon the cognitive representations used by persons, by determining the best conditions under which the appropriate representations are activated. Indeed such an approach appears to have significant didactic implications, in particular concerning the teaching of certain mathematical concepts.

Let us consider the following situation: there are three poker chips in a jar, two red ones and one white one. Two chips are picked out together, and the following two results «a red chip and a white chip are obtained» or «two red chips are obtained» have to be compared regarding their probability. While the correct expected answer is "there is more chance of obtaining a red chip and a white chip", previous studies (Lecoutre and Durand, 1988; Lecoutre, 1992) have shown that in this case most adults answer erroneously that the two results are equiprobable *«because it's a matter of chance»*. Analysis of the patterns of answers observed in a series of problems and of verbal reports has shown that random events are thought to be equiprobable «by nature». This representation, called the «equiprobability bias», is seen as an «implicit model» in the sense employed by Fischbein (Fischbein, 1987, 1989, 1994; Fischbein, et al., 1985). It can be compared to the «uniformity belief» reported by Falk (1992) for solving the «problem of three prisoners», that is to say people have a strong intuitive tendency to assume equal probabilities for the various available options. Additional convergent findings on the prominence of this intuitive belief can be found in a didactical study on college students' conceptions of randomness in which some students only consider a phenomenon to be random when all its outcomes are equally likely (Konold et al. 1991).

In the context of elementary arithmetic tasks solved by children, Fischbein et al. (1985) hypothesize on the nature and the role of implicit models involved in constructing the representation of this type of problem. These implicit models are constructed using various everyday-life experiences as a basis, and they are the origin of spontaneous representations. In some cases, these representations are inadequate for solving problems and are responsible for misconceptions. The models are considered as a basis for the interpretation of mathematical concepts which are deeply rooted despite the acquisition of formal concepts, and as structural schemata on which most intuition is based (Fischbein and Grossman, 1997; Fischbein and Schnarch, 1997; Lecoutre and Fischbein, 1998)In the problems involving drawing poker chips from a jar, it seems reasonable to assume that the implicit «equiprobability bias» guides the interpretation and explains the difficulties of perceiving the situation as an example of elementary combinatorial problems. We assumed that the main difficulties encountered by people were due to the same general phenomenon: the chance context of the «purely random» situation evokes an implicit model which is inadequate, and the misconceptions can be interpreted as a difficulty pertaining to gaining access to the adequate representation. More precisely in elementary uncertainty situations, the adequate combinatorial or logical representations are assumed to be available to most people, but not spontaneously evoked. Then the following question can be asked: what kind of experimental trick would be favorable to the activation of an adequate abstract representation of the situation?

Ross and Anderson (1982) suggested that effective experiences are those that require people to act upon their beliefs. As is demonstrated in recent research, learning would be enhanced by making students become aware of and confront their erroneous representations (delMas et al., 1998). In this framework, we set up an experiment with a learning phase aiming at examining whether learners can construct adequate representations of the situation if they are forced to see that their beliefs on earlier problems led to erroneous answers, and so to test the instructional effectiveness of active problem comparison on abstraction. During this learning phase, the subjects had to solve a series of four study problems which were pair-wise related by complementarity and equivalence. Indeed, two of the problems were isomorphic (« equivalent »), and the other two had a complementary structure. Thus respectively the four problems concerned:

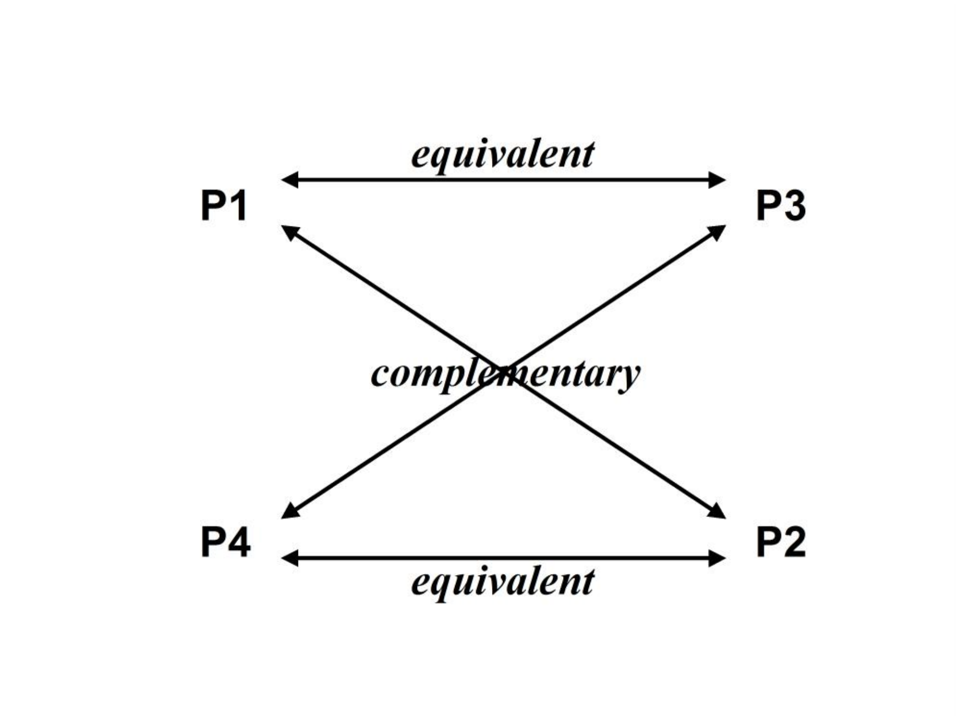
- Problem 1 (P1): one element drawn from the three;
- Problem 2 (P2): the two remaining elements after having drawn one element. Problem P2 will be called the «complementary problem» to P1 because its correct answer can be deduced by a complementarity relation from P1;
- Problem 3 (P3): the remaining element after having drawn two elements. Problem P3 is equivalent to P1 but described differently. P1 and P3 will be called «equivalent problems»;
- Problem 4 (P4): two drawn elements. This problem is complementary to P3 and equivalent to P2. This is the «classic» standard problem (Lecoutre, 1992).

The relations between these four problems are presented in Figure 1 and the wording of the four problems can be found in the method section.

Then a fifth « transfer » problem which was isomorphic to the fourth problem (P4) of the learning phase was given.

We predicted that if the students identified the relations, complementarity and equivalence relations, between the four study problems of the learning phase, , they would then succeed in constructing an adequate abstract representation, and a transfer to a fifth structurally isomorphic problem would then be observed.

Figure 1 - ***Relations between the four problems in the learning phase***



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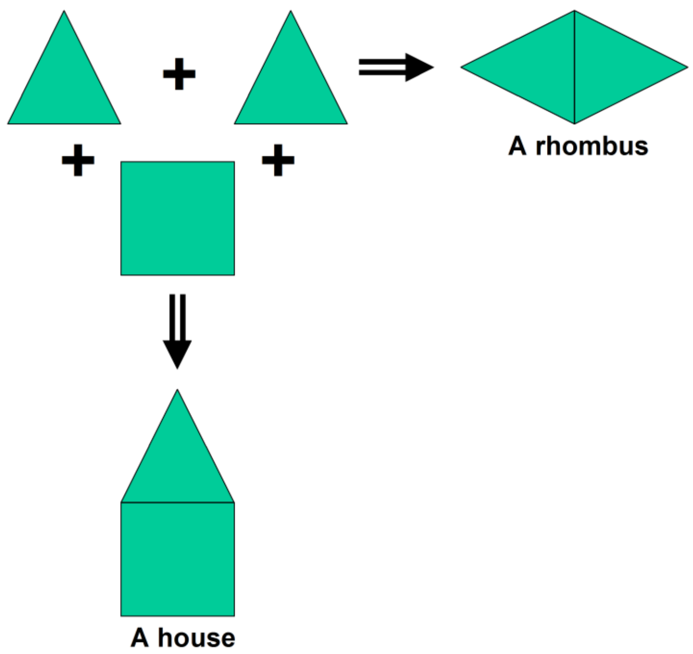
Learning phase

The four problems of the learning phase involved geometric figures, two triangles and a square. The size of the triangles was the same as the size of the square. The students were shown by means of visual animation on the computer screen that it was possible to construct either a house if a triangle and a square were drawn -therefore two ways of constructing a house-, or a rhombus if the two triangles were drawn -therefore only one way of constructing a rhombus- (see Figure 2).

Figure 2 - *The geometric figures*



Four problems reported in Table 1 were successively presented in the same order to all the participants. As referred to above, they are either «equivalent» (P1 and P3, P2 and P4), or «complementary» (P1 and P2; P3 and P4). Problem P4 corresponds to the «standard-problem» used in previous studies (Lecoutre, 1992).

Table1 - *The problems presented*

Learning Phase

Problem P1. Suppose that the three figures are in a jar. After having mixed them up, I draw one figure from the jar, and am interested in the drawn figure. The following two results are considered

Result R1: a triangle is obtained *(1)*
Result R2: a square is obtained

Do you think there is:

more chance of obtaining R1?
more chance of obtaining R2?
an equal chance of obtaining R1 and R2?
it is impossible to give an answer

Justify your response out loud

Problem P2. Suppose again that the three figures are in a jar. After having mixed them up, I have drawn one figure from the jar, and am interested in the two remaining figures in the jar after drawing this figure. The following two results are considered

Result R1: with the two remaining figures in the jar, it's possible to construct a house
Result R2: with the two remaining figures in the jar, it's possible to construct a rhombus

Do you think there is:

more chance of obtaining R1?
more chance of obtaining R2?
an equal chance of obtaining R1 and R2?
it is impossible to give an answer

Justify your response out loud

Problem P3. Suppose again that the three figures are in a jar. After having mixed them up, I have drawn two figures from the jar, and am interested in the remaining figure in the jar after drawing these two figures. The following two results are considered

Result R1: the remaining figure in the jar is a triangle
Result R2: the remaining figure in the jar is a square

Do you think there is:

more chance of obtaining R1?
more chance of obtaining R2?
an equal chance of obtaining R1 and R2?
it is impossible to give an answer

Justify your response out loud

Problem P4. (standard-problem). Suppose again that the three figures are in a jar. After having mixed them up, I draw two figures from the jar, and am interested in the two drawn figures. The following two results are considered

Result R1: with the two drawn figures, it's possible to construct a house
Result R2: with the two drawn figures, it's possible to construct a rhombus

Do you think there is:

more chance of obtaining R1?
more chance of obtaining R2?
an equal chance of obtaining R1 and R2?
it is impossible to give an answer

Justify your response out loud

Transfer Phase

Problem P5. Now suppose there are three chips are in a jar, two red chips and one white chip. After having mixed them up, I draw two chips from the jar, and am interested in the two drawn chips. The following two results are considered

Result R1: the two chips are a red chip and a white chip
Result R2: the two chips are two red chips

Do you think there is:

more chance of obtaining R1?
more chance of obtaining R2?
an equal chance of obtaining R1 and R2?
it is impossible to give an answer

Justify your response out loud

(1) In the present Table, in order to simplify the presentation of the results, result R1 is always the correct one; in the experiment, the assignment of the correct response to either results R1 or R2 is uncertain.

For each problem, the participants were asked to give their answers by choosing one of the following four possibilities: (1) more chance of obtaining R1; (2) more chance of obtaining R2 (3) an equal chance of obtaining R1 and R2 (4) it is impossible to give an answer.

Transfer phase

The fifth problem P5 involved poker chips, two reds and one white (cf. Table 1). It was structurally isomorphic to the fourth geometric-figures problem P4.

Procedure

The problems were displayed on a computer, and the students were tested individually. First they were shown how to answer by using the mouse, and were asked to familiarise themselves with the computer environment. Then they were asked to solve the problems, and were informed that there was no time constraint.

The five problems were presented with feed-back: in the case of a correct answer, the participant was given the following positive encouragement «Bravo, your answer is correct», and then the next problem was presented. In the case of an incorrect answer, the participant was told that his answer was incorrect and was asked to give another one; he was then given help with the possibility of going back to the correct answers to the previous problems. Therefore the participants could not solve the next problem until they had correctly answered the problem in question. Furthermore for each problem, the participants had to justify their answers (correct ones as well as incorrect ones) out loud.Experimentation times ranged from quarter of an hour to half an hour, depending on the individual.

Results

Two indications of behavior will be considered: the first answer given for each problem with the associated justification, and the pattern of answers for the successive problems.

Statistical analysis

Following the recent guidelines proposed of the American Psychological Association's Manuel (APA, 2001), interval estimates are given for each proportion (or the difference between proportions) of interest. We used the standard non-informative Bayesian procedures (see e.g., Lecoutre et al., 1995) in order to calculate 0.90 probability interval estimates. For inferences about proportions these intervals can be favorably compared to frequentist confidence intervals (Lecoutre and Charron, 2000; Brown, Cai and Das Gupta,2001). The Bayesian interpretation is that there is a 90% chance that the true proportion falls within the computed interval estimates. For each analysis, we give the observed percentage of interest, followed by the corresponding interval estimate in brackets.

1. The Learning Phase: Problems P1 to P4

The results obtained for each problem are reported in Table 2.

Problem 1. This problem which is related to drawing one object out of three, and considered, a priori, as the easiest one, led to the largest rate of correct answers (74%). The observed difference between this percentage and the mean percentage of correct responses to the other three problems is equal to 17%, with the interval estimate [3%,29%]. The justifications given by the students revealed that the representation activated in this case was an adequate representation in which they consider the number of elements. Thus for example, «*there are more triangles than squares, so there is more chance of drawing a triangle*». This «numbers model» was the only representation used in this case as in previous studies (see for example Lecoutre, 1984). However we must emphasize the surprisingly relatively high percentage of equiprobability responses (19% [11%,30%]) related to the activation of the «equiprobability bias».

Table 2 - *Percentages of answers and corresponding 90% probability interval estimates in brackets for each problem in the learning phase*

	Result R1	Equiprobable	Result R2	?
Problem 1	74% $_{(31)}$ [62%,84%]	19% $_{(8)}$ [11%,30%]	7% $_{(3)}$	0% $_{(0)}$
Problem 2	67% $_{(28)}$ [54%,78%]	29% $_{(12)}$ [18%,41%]	2% $_{(1)}$	2% $_{(1)}$
Problem 3	48% $_{(20)}$ [35%,60%]	36% $_{(15)}$ [25%,48%]	12% $_{(5)}$	5% $_{(2)}$
Problem 4	57% $_{(24)}$ [44%,69%]	21% $_{(9)}$ [13%,33%]	21% $_{(9)}$	0% $_{(0)}$

(n=42)

Problem 2. Problem P2, complementary to P1, shows quite a similar distribution of responses. Analysis of the bivariate data concerning P1 and P2 is reported in Table 3.

Table 3 - *Percentages of answers for P2 taking into account the answers for P1 (complementary problems) and corresponding 90% probability interval estimates in brackets*

	P1			
P2	Result R1	Equiprobable	Result R2	?
Result R1	64% $_{(20)}$ [50%,77%]	75% $_{(6)}$ [46%,92%]	67% $_{(2)}$	/
Equiprobable	29% $_{(9)}$ [17%,43%]	25% $_{(2)}$ [8%,54%]	33% $_{(1)}$	/
Result R2	3% $_{(1)}$	0% $_{(0)}$	0% $_{(0)}$	/
?	3% $_{(1)}$	0% $_{(0)}$	0% $_{(0)}$	/
	(/31)	(/8)	(/3)	(/0)

If we first consider the 74% participants who correctly solved P1, it must be noted that only 64% of them [50%,77%] also gave the correct answer first time round to P2. So the complementarity relation between P1 and P2 was not salient for many of the afore-mentioned participants. Furthermore 29% of them [17%,43%] gave an equiprobability response, yet this result points out the strength of the equiprobability bias. The answers of the students who activated the equiprobability bias in P1 reveal that even though 75% [46%,92%] gave the correct answer, 25% [8%,54%] also used it in P2. These findings show the difficulties encountered by the participants in reasoning about what remains in an urn after a draw, and so to realize the complementary status of situations.

Problem 3. In problem P3, equivalent to P1, the rate of correct answers is quite low (48% [35%,60%]), and is also the smallest one.

Analysis of the bivariate data concerning P1 and P3 which are equivalent problems, is reported in Table 4.

Table 4 - *Percentages of answers for P3 taking into account the answers for P1 (equivalent problems) and corresponding 90% probability interval estimates in brackets*

	P1			
P3	Result R1	Equiprobable	Result R2	?
Result R1	42% $_{(13)}$ [28%,57%]	62% $_{(5)}$ [34%,85%]	67% $_{(2)}$	/
Equiprobable	42% $_{(13)}$ [28%,57%]	12% $_{(1)}$ [2%,40%]	33% $_{(1)}$	/
Result R2	10% $_{(3)}$	25% $_{(2)}$	0% $_{(0)}$	/
?	6% $_{(2)}$	0% $_{(0)}$	0% $_{(0)}$	/
	(/31)	(/8)	(/3)	(/0)

Only 42% [28%,57%] of the 74% participants who had correctly solved P1 gave the correct answer to P3. This result shows that the equivalence relation was not really accessible and in any case not perceived as a salient feature by most students. An equal percentage of students (42%) gave the equiprobability response. These findings show that people reason differently about structurally isomorphic problems. These findings are consistent with those generally reported in research literature on analogical processing. Indeed the analogy between isomorphs is hardly perceived or used, unless a hint is given that the solution to a first problem might have something to do with the solution to a second one (see for example, Hayes and Simon, 1977 ; Keane, 1997).

In fact for this problem a little more than 15% of the subjects utilized an interesting representation called the «conditional model» (see Lecoutre, 1992) The reasoning was the following: knowing that in a pair of drawn elements, one of the two identical ones will inevitably be obtained (a triangle), consequently one element of each kind remains therefore there is an even chance for the second drawn object; as a result both results are equiprobable. Such a reasoning could be seen as a way to transform and cancel out the random aspect of the situation.

All these findings, like those previously reported for P2, clearly demonstrate that for most people it is difficult (or even impossible) to reason about what remains after a draw. Indeed in most every-day life situations we are led to reason about what has been drawn (in lotteries for example) and hardly ever about what remains. The activation of this model could be enough to «mask» the perception of the equivalence «drawn element (P1)/remaining element (P3)». This result could be interpreted as the expression of the general analogical transfer process. Indeed in this case the general knowledge constructed by experience of lotteries is imported into the new situation in which such knowledge is irrelevant.

Problem 4. 57% [44%,69%] of the participants correctly solved this problem. Analysis of the bivariate data concerning P4 and P2 (equivalent problems) on the one hand, and of P4 and P3 (complementary problems) on the other hand is reported respectively in Tables 5 and 6.

Table 5 - *Percentages of answers for P4 given taking into account the answers for P2 (symmetrical problems) and corresponding 90% probability interval estimates in brackets*

	P2			
P4	Result R1	Equiprobable	Result R2	?
Result R1	57% $_{(16)}$ [42%,72%]	50% $_{(6)}$ [28%,72%]	1% $_{(1)}$	1% $_{(1)}$
Equiprobable	25% $_{(7)}$ [14%,40%]	17% $_{(2)}$ [5%,39%]	0% $_{(0)}$	0% $_{(0)}$
Result R2	18% $_{(5)}$	33% $_{(4)}$	0% $_{(0)}$	0% $_{(0)}$
?	0% $_{(0)}$	0% $_{(0)}$	0% $_{(0)}$	0% $_{(0)}$
	(/28)	(/12)	(/1)	(/1)

Only a little more than half of the 67% participants who had correctly solved P2 gave the correct answer for P4 (57% [42%,72%]). This result is to be compared to the one previously reported for the other pair of equivalent problems (P1 and P3): for many participants, the equivalence relation did not appear to be a salient feature.

Table 6 - *Percentages of answers for P4 taking into account the answers for P3 (complementary problems) and corresponding 90% probability interval estimates in brackets*

	P3			
P4	Result R1	Equiprobable	Result R2	?
Result R1	60% $_{(12)}$ [42%,76%]	53% $_{(8)}$ [33%,73%]	60% $_{(3)}$	50% $_{(1)}$
Equiprobable	25% $_{(5)}$ [12%,43%]	13% $_{(2)}$ [4%,32%]	20% $_{(1)}$	50% $_{(1)}$
Result R2	15% $_{(3)}$	33% $_{(5)}$	20% $_{(1)}$	0% $_{(0)}$
?	0% $_{(0)}$	0% $_{(0)}$	0% $_{(0)}$	0% $_{(0)}$
	(/20)	(/15)	(/5)	(/2)

Only 60% [42%,76%] of the 48% students who had correctly solved P3 gave the correct answer for P4. This result is to be compared to the one obtained for the other pair of complementary problems (P1 and P2). It reveals yet again that the complementarity relation does not seem to have been used by most students when answering.

Thus, at the end of the learning phase, it appears that the instructional intervention used was not very effective : the learning effect was clearly below what might have been expected. To go into more detail in the interpretation of our findings, it is interesting to examine the sequences of four responses successively given from P1 to P4 during the learning phase. For example, «+R1 = +R2 =» is the pattern of a participant who answered «+R1» (correct response) for P1, «=» (equiprobability response) for P2, «+R2» (more chance of obtaining R2) for P3, and «=» for P4. Individual patterns were very heterogeneous from one participant to another: 26 different patterns (for 42 participants) were registered, amongst which 18 of them were only observed once. It is likely that this great heterogeneity is due to the fact that for most participants each problem was treated as a new problem, independently of the preceding ones. In the case of a first incorrect answer, only slightly less than a third of the participants (32%) asked to go back to the correct answers to the previous problems, which were rarely spontaneously perceived as relevant for solving a new problem.All these findings are convergent, and support the conclusion on the absence of construction of an adequate representation of the situation for many students at the end of the learning phase. This is due to the fact that the complementarity as well as the equivalence relations were not used to solve the problems because they were not salient features and therefore were not an efficient means of assistance for many participants.

2. The Transfer Phase: Problem P5

The findings for P5 are reported in Table 7.

Table 7 - *Percentages of answers for P5 and corresponding 90% probability interval estimates in brackets*

	Result R1	Equiprobable	Result R2	?
Problem 5	64% $_{(27)}$ [52%,75%]	10% $_{(4)}$ [4%,19%]	24% $_{(10)}$	2% $_{(1)}$

(n=42)

(1) Only 64% [52%,75%] of the participants gave the correct answer for P5, and this

is quite a low percentage which does not appear to be very different from the 57% observed for Problem 4. It is important to recall here that in the learning phase the problems were systematically presented until the participants gave the correct answer. Therefore all the participants knew all the solutions, and in particular the solution for P4, which is an isomorphic problem. Taking into account this fact it seems very surprising that more than a third of them still gave an incorrect answer for P5. These findings reveal that knowing the correct answers is not enough to be able to construct a correct representation. Indeed as long as the participants did not register the relations between the problems (complementarity and equivalence relations), they did not succeed in constructing the correct abstract representation of the structure of the problems.

(2) Analysis of the bivariate data concerning P5 and P4 (isomorphic problems) is reported in Table 8.

Table 8 - *Percentages of answers for P5 taking into account the answers for P4 (isomorphic problems) and corresponding 90% probability interval estimates in brackets*

	P4			
P5	Result R1	Equiprobable	Result R2	?
Result R1	79% $_{(19)}$ [63%,90%]	44% $_{(4)}$ [21%,70%]	44% $_{(4)}$	/
Equiprobable	12% $_{(3)}$ [5%,27%]	11% $_{(1)}$ [2%,36%]	0% $_{(0)}$	/
Result R2	8% $_{(2)}$	33% $_{(3)}$	56% $_{(5)}$	/
?	0% $_{(0)}$	11% $_{(1)}$	0% $_{(0)}$	/
	(/24)	(/9)	(/9)	/

Quite a high transfer rate was observed, as 79% [63%,90%] of the participants who correctly answered P4, gave a correct answer first time round for P5.

This result appears compatible with the hypothesis according to which a transfer to isomorphic problems would be easier when the subjects succeed in processing the relations between the problems during the learning phase, and then succeed in constructing an abstract representation of the situation.

An exam of the individual patterns of the 79% subjects who transferred shows that for most of them (80%), there is no or at the most only one incorrect response during the learning phase. So most subjects who performed correctly at least three of the four training problems also solved correctly the transfer problem.

Conclusion

Our findings highlight three main points. (1) As far as the learning phase is concerned, only a little more than half of the participants (57%) correctly solved the classic standard problem (P4), whilst 21% of the participants gave the incorrect equiprobability answer for this problem («equiprobability bias»). Once again these findings point out the difficulties of this kind of problem despite the fact these problems are often considered self-evident. (2) The instructional intervention used in the learning phase was not very effective because for most students the two relations between the four training problems, complementarity and equivalence, which were supposed to help them for the construction of an adequate representation, were not perceived as salient features and therefore were not an efficient means of assistance. Indeed all four types of problems were quite independent for many students. These findings show that (a) people reason differently about structurally isomorphic problems and (b) fail to realize the complementary status of situations. (3) Transfer occurred only for the subset of subjects who performed correctly on the training problems of the learning phase. . O

In order to emphasize the salience of the relations between the four problems in the learning phase, two axes are being explored. (1) The effects of the order of presentation of the problems. Let us recall that in the present study, the equivalent problems were never consecutive unlike the complementary problems, and a crucial question can be formulated as follows: is there an order which is more conducive to constructing one representation than another? A series of new experiments in which the order of presentation is systematically counterbalanced, is being designed in order to answer this question. (2) The effects of the type of feedback and instructions. For instance the subjects could be asked to not only justify their own answers as in the present study, but also the correct answers to each problem. Indeed much recent work shows that the performance of some students can be improved when they are asked to systematically «self-explain» (see Robertson, 2000). Furthermore it would also be interesting to study the effects of providing the students explanation concerning the relations between the four problems proposed in our work at a level of generality which would be sufficient enough to enable them to first of all construct the adequate abstract representation, and then secondly to adapt the reasoning to suit various isomorphic problems. Such a result has recently been observed by Robertson (2000) with algebra word problems taken from Reed et al. (1985).

Nevertheless the results obtained in the present study already appear to have some significant implications in the teaching of certain mathematical concepts. Indeed they suggest the value, if not the necessity, of attempting not only to « exhibit » the representations used by the subjects, but only to act upon these representations. With this purpose in mind, situations in which the students are led to construct by themselves the adequate representations are given priority.

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